Please answer TWO of the following three questions

1. Algebra

Consider a sequence \( \{a_n\} \). The sum from the first term to the \( n \)th term of the sequence is denoted \( S_n \), and satisfies:

\[
S_n = 2a_n - n \times 2^{n+1} \quad (n = 1, 2, 3, \ldots)
\]

a) Calculate \( a_1 \) and \( a_2 \).

b) Express \( a_{n+1} \) in terms of \( a_n \) and \( n \) by calculating \( S_{n+1} - S_n \).

c) \( b_n \) is defined as follows:

\[
b_n = \frac{a_n}{2^n}.
\]

Express \( b_n \), then \( a_n \) in terms of \( n \).

2. Geometry

In this question, \( x \) and \( y \) are real numbers.

a) Sketch a graph of the following equation:

\[
(x - \frac{1}{2})^2 + (y - \frac{1}{2})^2 = \frac{1}{2}.
\]

b) Sketch the region that satisfies the following inequality:

\[
x^2 + y^2 \leq |x| + |y|.
\]

c) Find the area of the region described in (b).

3. Calculus

Consider the function \( f(x) \), defined as follows:

\[
f(x) = \int_a^x (t - a)(t - b) \, dt,
\]

where \( a \) and \( b \) are real constant values.

a) Evaluate the function \( f(x) \) in terms of \( a \) and \( b \).

b) Find the values of \( a \) and \( b \) that satisfy the following three conditions on \( f(x) \):

i. \( f(x) \) has a maximum or minimum at \( x = \frac{1}{2} \),

ii. \( f(a) - f(b) = \frac{1}{6} \),

iii. \( f'(0) > 0 \).

c) Determine all maxima and minima of \( f(x) \), and sketch the function.